Instructions:

1. Duration is 4 hours.
2. Use white plane A4 papers for your work and write your name on each paper you submit.
3. Proofs should be provided for all lemmas used and all theorems used should be properly named and stated.
4. Ask questions about any problem you do not understand within the first 30 minutes of the exam.

Problem 1: $ABCD$ is a quadrilateral with,

$$|AB| = |AD|, \cos\left(\frac{1}{2}\angle BAD + \angle DCB\right) = 0 \text{ and } \frac{1}{\sqrt{3}} \leq \frac{|AC|}{|AB|} \leq \sqrt{3}.$$

What is the minimum possible value of $\angle BAD$?

Problem 2: Find all triples of primes (p, q, r), such that

$$\begin{cases} p^q r^2 + q^r p^2 = r^p q^2 + 13 \\ pr^2 + qp^2 = rq^2 + 73 \end{cases}$$

Problem 3: In a gathering of people, if 2 people know each other, we say there is a ***connection*** between them. ***Connection*** is mutual. If 3 people all know each other, we say they are a ***3clique***. Given that there are 7 people in a gathering, determine with proof, the minimum number of ***connections*** that guarantees the existence of a ***3clique***.

Problem 4: Find all functions $f: \mathbb{Q} \longrightarrow \mathbb{R}$ such that there exist functions $g: \mathbb{Q} \longrightarrow \mathbb{R}$, $h: \mathbb{Q} \longrightarrow \mathbb{R}$ with $f(1) = 1$, $h(0) \neq 0$ and

$$g(x + y) + g(x - y) - 2g(x) = h(x)f(y) \forall x, y \in \mathbb{Q}$$